Parton Branching in Color Mutation Model

Rudolph C. Hwa[1] and Yuanfang Wu[2]
Institute of Theoretical Science and Department of Physics
University of Oregon, Eugene, OR 97403-5203

Abstract

The soft production problem in hadronic collisions as described in the eikonal color mutation branching model is improved in the way that the initial parton distribution is treated. Furry branching of the partons is considered as a means of describing the nonperturbative process of parton reproduction in soft interaction. The values of all the moments, $\langle n \rangle$ and C_q, for $q = 2, \cdots, 5$, as well as their energy dependences can be correctly determined by the use of only two parameters.

1 Introduction

Recently a color mutation model [1] (hereafter referred to as I) was proposed for soft production in hadronic collisions. It is called ECOMB, which stands for eikonal color mutation branching. Unlike the string model, it starts out with the existence of partons as in the parton model, and describes the evolution process in which a large color neutral cluster breaks up into smaller and smaller clusters in successive fission subprocesses. Its primary achievement is in the capability of fitting the intermittency data of NA22 [2], as no other dynamical model has. There is, however, one defect in that version of ECOMB. The energy dependence of the initial parton multiplicity in the range $10 < \sqrt{s} < 100$ GeV is encoded in an *ad hoc* way, since the emphasis in I has been placed in the movements of the partons in the rapidity and color spaces as they undergo color mutation, not in their generation. In this paper we remove that defect by focusing on the branching of partons that leads to the initial parton distribution considered in I.

It should be made clear that there are two main parts in the overall process of soft production as described by ECOMB. One is how the quarks and gluons in the parton model evolve and end up as hadrons. That part has been treated in I. The other part is concerned with the number of partons at the beginning of the evolution process and how it increases with the collision energy. It is this part that we treat now.

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[1]E-mail address: hwa@oregon.uoregon.edu

[2]Permanent Address: Institute of Particle Physics, Hua Zhong Normal University, Wuhan, China

Both parts are nonperturbative, and both involve branching, but in different senses. In I the color mutation process leads to the branching of color neutral clusters that terminates in hadron formation. The total number of partons do not change in that process. To be described below is the branching of partons as a result of the collision between incident hadrons. In that process the total number of partons increases with energy; it accounts for the logarithmic increase of the observed average multiplicity with s. But even at a fixed s, the parton number increases with the evolution parameter in the branching process so that from a limited number of valence quarks, a large number of gluons and sea quarks can be generated by the interaction. It is the s dependence of that evolution parameter that is incorporated in a natural way with the parton branching process that results in the s dependence of the hadron multiplicity.

2 Parton Branching

In I it is shown how the eikonal function $\Omega(R)$ forms the basis for the geometrical-scaling description of the multiplicity distribution

$$P_n = \int dR^2 \left(1 - e^{-2\Omega(R)}\right) \mathbf{Q}_n(R) \quad , \tag{1}$$

where $\mathbf{Q}_n(R)$ is the probability of producing n particles at the scaled impact parameter R. It is further described in I how $\mathbf{Q}_n(R)$ is determined in an evolution process, denoted symbolically by $\mathcal{E}_{m \to n}$, such that

$$\mathbf{Q}_n(R) = \mathcal{E}_{m \to n} \left\{ Q_m(R) \right\} \quad , \tag{2}$$

in which m partons evolve by color mutation into n hadrons. In I the number of partons is denoted by ν, a symbol which we now reserve for the number of wounded nucleons in a pA collision, a subject of a separate investigation. The probability of producing m partons before the evolution, $Q_m(R)$, at R is related to the probability of having m partons in a μ-cut Pomeron by

$$Q_m(R) = \sum_{\mu=1}^{\infty} \pi_\mu(R) B_m^\mu / \sum_{\mu=1}^{\infty} \pi_\mu(R) \quad , \tag{3}$$

where

$$\pi_\mu(R) = \frac{[2\Omega(R)]^\mu}{\mu!} e^{-2\Omega(R)} \quad . \tag{4}$$

In I, B_m^μ is assumed to be Poissonian with an s-dependent parametrization of the average $\bar{m}$. We now improve on this portion of the formulation by incorporating the branching dynamics of the Furry process.

Our problem is the determination of the number of partons that are excited by the collision process before they undergo color mutation, leading eventually to the observed hadrons. That number, m, must increase with energy and should depend on the impact parameter R. By the transformation given in (3), the dependence on R is transferred to the

dependence on μ, which denotes the number of cut Pomerons. A Pomeron represents a non-planar diagram with vacuum quantum number exchanged and can be expressed more readily in terms of the constituents of the incident particles [3], instead of the impact parameter of the collision geometry. Thus B_m^μ is the appropriate quantity to study when we ask how the constituents of the incident particles increase in number through successive emissions to arrive at m partons.

The production of partons in hadronic collisions has been a subject of long standing. The parton cascade model (PCM) [4] is among the more recent attempts, but it is based entirely on perturbative QCD whose validity for soft processes is known to be unreliable. Indeed, the comparisons of the predictions of PCM with the data are all for $\sqrt{s} > 200$ GeV, whereas the data for soft production only are for $\sqrt{s} < 100$ GeV. Thus we reject the use of perturbative QCD for soft interaction. Nevertheless, we want to consider the proliferation of partons by a branching dynamics, which is the universal way that the growth of a population has been successfully described in many fields. Toward that end we adopt the Furry process, originally proposed by cosmic-ray showers [5], as the mechanism that generates the partons. Based on the assumption that the branching process is purely statisitcal, the evolution equation that a Furry distribution, F_j^k, satisfies is

$$\frac{d}{dt}F_j^k = (j-1)F_{j-1}^k - jF_j^k \quad , \tag{5}$$

which belongs to the generic form of a stochastic branching equation [6]. It states that the rate of change of F_j^k is due to the gain from the emission by any one of the $j-1$ partons at a previous time step and the loss from the j partons state when one of them emits one more. The time variable t is normalized in such a way that the coupling strength does not appear explicitly in (5). Since (5) does not rely on the smallness of any coupling, it is not perturbative. The label k fixes the initial condition in that at $t = 0$ there are k emitters, i. e., $F_j^k(t=0) = \delta_{jk}$.

The solution of (5) is [6]

$$F_j^k(w) = \frac{\Gamma(j)}{\Gamma(k)\Gamma(j-k+1)}\left(\frac{1}{w}\right)^k\left(1 - \frac{1}{w}\right)^{j-k} \quad , \tag{6}$$

where w is the evolution parameter, $w = e^t$. From the generating function of F_j^k, it can easily be shown that the average j is $\bar{j} = wk$. Thus $F_j^k(w)$ is specified by

$$w = \bar{j}/k \quad , \tag{7}$$

which simply states that the extent of evolution is related to how much $\bar{j}$ is increased above the initial number k. In particle physics w is more meaningful than t; moreover, $\bar{j}$ increases logarithmically with energy.

It is of interest to point out that the Furry distributions can be related to the negative binomial distribution, P_n^k, by [6]

$$F_j^k\left(\frac{\bar{j}}{k}\right) = P_{j-k}^k\left(\frac{\bar{j}}{k} - 1\right) \quad . \tag{8}$$

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The latter distribution has been used extensively to fit the high-energy data on multiplicity distributions [7]; however, it is applied directly to the produced hadrons with k used as an adjustable parameter, which is shown phenomenologically to decrease with energy. In contrast, our consideration of the Furry distribution is for the branching of partons with k having a more theoretical basis than being a free parameter.

Our immediate task is to relate k to μ, since F_j^k is to be used in place of B_m^μ in (3). Consider the high-energy regime where the Pomeron dominates. A Pomeron is a nonplanar diagram with the exchange of a cylinder, not a planar ladder. A nonplanar diagram is possible only if more than one constituent line in each incident particle is involved in the exchange of a ladder. The simplest case of one-cylinder exchange involves two partons in each particle, (a_1, a_2) in particle a and (b_1, b_2) in particle b, say. A ladder between a_1 and b_1, and another between a_2 and b_2, together form a nonplanar diagram. Cutting the rungs of these two ladders gives rise to a $\mu = 1$ cut-Pomeron. Each half of the cut diagram has four initial partons connected by two parallel combs of partons. The t-channel cut diagram can be viewed in the s channel as successive emissions of partons. Thus we can identify the process as having been initiated by four partons (i. e., $k = 4$ initial emitters) and ending in j total number of final partons. The fluctuation of j around $\bar{j}$ is to be specified by F_j^k, and $\bar{j}$ increases linearly with the total rapidity opened up by the collision.

When there are more than one cut Pomeron, then the number of initial emitters must increase accordingly in order that there can be more cylinders in the nonplanar diagram, one for each additional cut Pomeron. For that reason we set

$$k = 4\mu \quad . \tag{9}$$

At lower energy, say $\sqrt{s} < 20$ GeV, even though geometrical scaling may still be valid to justify the use of an s-independent $\Omega(R)$ to describe σ_{tot} and σ_{el}, the reduced phase space limits the multiplicity of particles produced. It means that (9) needs to be modified at low energies by a factor that suppresses high k at low s, as μ is summed to high values in (3).

To make contact with B_m^μ, it is necessary to relate j to m, in addition to relating k to μ. So far in this section we have referred to partons without specifying whether they are quarks, antiquarks or gluons. We now assert that in soft interaction we only have to consider the Furry branching of the gluons for the following reasons. (a) There are far more gluons than sea quarks. (b) The valence quarks are not to be counted among the partons that are to hadronize in the central region (as calculated by ECOMB), since they produce the leading particles in the fragmentation region. (c) The gluons first emitted by the valence quarks can initiate the Furry branching process. (d) It is implicit in Eq. (5) that all emissions are identical in character, thus implying only one type of coupling, i.e., three-gluon coupling. (e) For $\mu = 1$, the successive gluon emissions from four gluons form two non-planar ladders; for $\mu = 2$ there are eight initial emitters, and so on. Thus j is to be identified with the number of gluons at the end of Furry branching. Sea quarks are obtained by the conversion of those gluons to $q\bar{q}$ pairs.

In I when m partons (denoted by ν there) are considered for the evolution of the color mutation process, it has been explicitly specified that those partons are quarks and antiquarks that form two overlapping color-neutral initial clusters. Thus before that evolution process begins, all gluons are to be converted into $q\bar{q}$ pairs in a procedure, called the "saturation of the sea", used in the original recombination model [8]. It is in that way that the

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hadronization of gluons is taken into account without the formation of any glueballs. This saturation of the sea is for accounting the partons produced in the central region; dressing of the quarks by gluons in the nonoverlapping region of the hadronic collision leads only to the leading particles in the fragmentation region, which is not our concern here. How quarks turn into hadrons in the central region has been treated in I. Here we simply count each gluon as a $q\bar{q}$ pair, so

$$m = 2j \quad . \tag{10}$$

Taking together (9) and (10), we now have

$$B_m^\mu = \sum_{j,k} F_j^k \, (\bar{j}/k) \, \delta_{k,4\mu} \, \delta_{m,2j} \quad . \tag{11}$$

We use this formula for all $\sqrt{s} > 20$ GeV. At lower s the inability to produce many particles due to energy conservation (even though the masses of hadrons are not considered explicitly here or in I) is effected by cutting off k at 16. That means that for $10 < \sqrt{s} < 20$ GeV we replace $\delta_{k,4\mu}$ in (11) by $\delta_{k,4\mu} \, \theta(4 - \mu) + \delta_{k,16} \, \theta(\mu - 4)$. Since $\pi_\mu(R)$ is small for $\mu \geq 4$ at all R (see Fig. 1 in I), only a small part of the calculation is affected by this correction. The important point is to recognize that Eqs. (1) - (4), supplemented by (11) completely specify the formalism of the problem in a manner appropriate for Monte Carlo calculation. The simulated value of m is then used as the initial number of partons (i.e., quarks and antiquarks) that undergo color mutation and hadronization as described in ECOMB. The only adjustable parameters to be used are in the s dependence of $\bar{j}$, as will be discussed in the next section.

3   Multiplicity Distribution

The multiplicity distribution of the final state hadrons is calculated on the basis of Eqs. (1) - (4) and (11). Among them, (2) involves a complicated process that determines where those hadrons go in the phase space, an aspect of the problem that does not concern us here. The hadron multiplicity n is essentially half the parton multiplicity m, since only the recombination of q and $\bar{q}$ into mesons is considered. However, the mesons may be resonances, whose decays make n greater than $m/2$. For that reason ECOMB must be used in (2) to determine P_n, except that B_m^μ in I is now replaced by (11).

In Eqs. (1) - (4) there is no explicit dependence on s. The only place where s enters is in $\bar{j}$ in (11), since the extent of parton branching depends on s. The phenomenology of multiplicity distribution in the range $10 < \sqrt{s} < 100$ GeV confirms KNO scaling [9], which places a severe constraint on any model on multiparticle production. It requires that $\langle n \rangle P_n$ plotted against $n/\langle n \rangle$ be independent of s, even though $\langle n \rangle$ itself increases logarithmically with s. The s-independence of the equations (1) - (4) by no means guarantees KNO scaling. Indeed, it has been a challenge in I to generate ℓn s increase of $\langle n \rangle$, while keeping the moments

$$C_q = \langle n^q \rangle / \langle n \rangle^q \tag{12}$$

roughly constant. We now meet that challenge here with $\bar{j}$ being the only s-dependent evolution parameter in the Furry branching.

We use the parametrization

$$\bar{j} = c_0 + c_1 \ell \mathrm{n} \sqrt{s} \quad , \tag{13}$$

where c_0 and c_1 are our only two free parameters, adjusted to fit $\langle n \rangle$ vs $\ell \mathrm{n}\, s$. The result is shown in Fig. 1, for which we have used

$$c_0 = -7 \quad , \qquad c_1 = 6 \quad . \tag{14}$$

Evidently, the fit of $\langle n \rangle$ is very good. Its success may not be surprising given two free parameters. However, what is striking is that without any further adjustment of any other parameters the calculated values of C_q turn out to be essentially constant for $\sqrt{s} > 10$ GeV. Moreover, their values agree remarkably well with the ISR data [10], also shown in Fig. 1. It should be noted that for the two lowest-energy points a correction to (11) as remarked below that equation has been used.

Given how well our calculated C_q moments agree with the data, it should not be a surprise to see the multiplicity distribution exhibiting excellent KNO scaling. That is shown in Fig. 2 where $\psi(z) = \langle n \rangle P_n$ is plotted against $z = n/\langle n \rangle$ for ten different s values. The curves are essentially indistinguishable from one another. We emphasize that this scaling behavior is a highly nontrivial result and is not guaranteed by our geometrical scaling formalism.

4   Conclusion

The part of ECOMB described in I that specifies the initial parton number m before color mutation is now amended in this paper. We have used Furry branching to determine the increase of the number of gluons from k to j. By using only two parameters to describe the s dependence of $\bar{j}$, we have been able to determine $\langle n \rangle$ and C_q, for $q = 2, \cdots, 5$, all of which agree with the ISR data over the whole range $10 < \sqrt{s} < 70$ GeV [10]. Together with its capability to fit the intermittency data [2] as shown in I, the modified version of ECOMB has achieved in fitting all essential data on soft production in hadronic collisions.

With the s dependence under control we can now consider the extension to higher energies where the hard production of minijets is also important in addition to soft production. It is a subject that is a natural extension of the present work. Furthermore, the generalization to pA and AA collisions will eventually lead this line of work to the study of collision processes that will take place at RHIC. In such processes the proper treatment of both the soft and the hard production of particles is essential.

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Figure Captions

Fig. 1   Average charge multiplicity $\langle n \rangle_{ch}$ and the moments C_q as functions of energy. The data are from [10]; the lines are calculated results.

Fig. 2   Calculated KNO distributions for the 10 $\sqrt{s}$ points in Fig. 1. $\psi(z) = \langle n \rangle P_n$ and $z = n/\langle n \rangle$.